

Teacher training and quality of mathematics education

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Abstract: The problem of teacher training in mathematics at the secondary education level is recurrent, as shown in various publications on the history of education. Therefore, it is difficult to understand why so little progress has been made, despite the many complaints and proposals. In this sense, the aim of this study was to analyze the relationship between the training and the processes of improvement in the teaching of mathematics teachers. Currently, content knowledge is almost the only professional reference for most teachers. This is despite the fact that no one disputes the fact that teachers also need to have other knowledge of Didactics of Mathematics and knowledge derived from teaching practice. In this sense, it is important to establish a stable, rigorous and coherent institutional framework between university and non-university institutions involved in initial and continuing teacher training, which will make it possible to seriously and rigorously address the problems we have discussed.

Keywords: Proposals, mathematics, education, knowledge

1. Introduction

Teacher training has been one of the factors linked to the quality of education. The results of widely known national and international evaluations indicate the existence of alarming deficiencies in student performance in science, reading and mathematics, from elementary to high school [1]. More than 20 years ago, Aguilar, Guevara, Latapí and Cordera (1992) stated that Mexican education was of very low quality, suffered from school and social inefficiency, had disarticulation between educational levels, the contents taught were irrelevant, the evaluation was inefficient, among other elements, and among



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the probable causes of this, a weak teacher training was pointed out [2].

Mathematicians have mathematics itself and its development as an object of study; engineers use mathematics as a tool for the design of civil works, such as bridges, buildings, houses, among others. Mathematics teachers, on the other hand, have as their object of study the processes of teaching and learning mathematics [3]. The relationship of mathematics with these professionals defines their professional area; in the former, it is the discipline as such, in the latter, it is its use as a tool for design, and in the latter, it is to make it the object of learning [4,5]. Therefore, we propose that a mathematics teaching professional has as the object of his or her profession precisely the teaching and learning of mathematics, and his or her essential objective is to promote learning. So, how can we train mathematics teachers to foster learning in their students?

Let us begin by clarifying what is meant by profession. The word profession comes from the Latin *professio-onis*, which means action and effect of professing. It is used with different meanings: employment, faculty or trade that everyone has and exercises publicly [6,7]. Professions are occupations that require specialized knowledge, high-level educational training, control over the content of the work, self-organization, self-regulation, altruism, a spirit of service to the community and high ethical standards [8,9].

It is generally accepted that a profession is a specialized activity within society, and the person who performs it is called a professional. Fernández (2003) agrees with this characterization and suggests six fundamental features of the profession: specific non-trivial knowledge, continuous technical progress, a critical-scientific foundation, self-perception of the professional, a certain level of institutionalization and social recognition [4,5]. Specific non-trivial knowledge, of a certain complexity and difficulty of mastery, marks the difference between those who practice the profession and those who do not, precisely because the former have this knowledge and the latter do not. In our case, specific knowledge and power refer, at the same time, to mathematical, didactic and teaching knowledge. This set of knowledge, integrated and put to good use, is supposed to be exercised by those who are capable of fostering learning in their students [10,11].

Continuous progress of a technical nature refers to the fact that scientific-technical conceptions about specific knowledge are changing, given the scientific progress of the disciplinary field of teaching sciences and the needs and possibilities of service to society. The conceptions about the teaching of mathematics have changed from positions and practices close to the behaviorism of the last century to the current constructivist positions. The critical-scientific foundation on which it rests and finds justification, has evolved from the positions of teaching mathematics as an art to the positions of those of us who claim this activity as an essential part of a scientific discipline: Educational Mathematics [12,13]. Therefore, nowadays the study of teacher training and professional practice from a scientific point of view is increasingly growing with the purpose, on the one hand, of identifying regularities and achieving more and more objective explanations that can configure a systematic body of knowledge, and on the other hand, that this body of knowledge can impact the educational reality to improve it [4,14,15].

Finally, social recognition comes from the prestige earned by the guild in society, and this, in turn, is influenced by society's level of satisfaction with the results of the work of mathematics teaching professionals. With such poor results in mathematics, I do not believe that Mexican society is satisfied or has a high esteem for the work of those of us who dedicate ourselves to the professional practice of mathematics teaching [3,5,16].

2. Professional training

Our approach coincides with the previous one, however, it is broader and more comprehensive, based on the experience we have gained in the training of teachers through the degree in Educational

Mathematics at the Autonomous University of Guerrero since 1986. We start from the fact that the general objective of professional training is to develop competencies so that future teachers can propitiate or produce mathematical learning [17]. For this to be possible, it is necessary to master mathematical knowledge, to know how students learn and, on these bases, to be able to use or design the methods, procedures and didactic means that make learning possible. Therefore, the training of mathematics teachers is articulated on the basis of three fundamental areas: mathematical, pedagogical and teaching [18–20].

The basic disciplinary training corresponds to the mathematical area, which is one of the foundations of the professional's work. Its objective is the development of competencies in the domain of university mathematics, emphasizing at the same time the development of particular competencies such as: abstraction, validation and applications, mainly in problem solving [21]. This area is intended to form a deep and updated vision of the nature of mathematics, its content and its meaning, in order to face the problems that transposing it to school brings with it. By virtue of this, it is composed of two fields: one disciplinary and the other epistemological [22].

The disciplinary field is structured around four axes that permeate the curriculum longitudinally and are similar (with due proportions) to the curriculum of secondary schools: Algebra, Geometry, Analysis, Statistics and Probability. The curricular axes attempt to confront the fragmentation and disconnection of content pointed out by Burril (2000), putting in their place the development of ideas and forms of mathematical thinking that permeate the entire curriculum. Under this premise, the aim is to develop in future teachers algebraic thinking, geometric thinking, variational thinking and language, stochastic thinking and logical-deductive thinking [23,24].

On the other hand, the epistemological axis has the purpose of analyzing how the main mathematical ideas are born, how they are developed and how they are validated. This field finds its relevance in the constructivist thesis of the existence of a certain parallelism between the phylogenesis and ontogenesis of mathematical knowledge. That is to say, just as humanity developed this type of knowledge, it is expected that today's students can develop (or rather reconstruct) it in school conditions with the help of the teacher [25,26].

The pedagogical area is the curricular space dedicated to knowing how students learn and, on this basis, to be able to use or design the methods, procedures and didactic means that make learning possible. It is organized into three training fields: psychological, didactic and technological. The first aims at knowing how students learn, what obstacles they face when constructing knowledge and how to overcome them, what mathematical ways of thinking are necessary for understanding, what understanding is and by what means students can be helped to provoke it, what mathematical intelligence is, what is the difference between habitual learning and intelligent learning, among other aspects [27,28].

On the other hand, the didactic field has as its objective (based on the psychological knowledge of the learner) the use and design of teaching strategies, methods and procedures that make learning possible. In this field, the analysis and discussion of core issues associated with the planning, execution and evaluation of the school mathematics curriculum takes place, such as: why teach mathematics, what mathematics to teach, how to teach mathematics, what, how and when to evaluate the learning of mathematics? The first two questions refer to the objective of teaching mathematics in school and to the forms or approaches of how to organize school mathematical content [3–5]. The third refers to the knowledge, design and innovation of methods and procedures for teaching mathematics in concrete situational conditions. The fourth question refers to the knowledge, design and innovation of methods and procedures for the evaluation of school mathematics. We start from the assumption that didactics is the science of teaching and its objective is to develop skills to support and guide the process of teaching mathematics from a scientific point of view [29,30].

3. Professor training

The teaching area is the essential part of the professional's training and it is where all the training areas are integrated. Teaching is considered as a profession that has to solve certain problems immediately (order, questions, unforeseen situations, for example), although for others it requires some time for reflection before making decisions (planning, organization, selection of resources, to mention a few). In any case, this is a practical profession, so professional competence cannot be reduced to its theoretical dimension. It presupposes the substantial improvement of educational practices in the mathematics classroom and requires teacher training that has firm links with both sides: with the practical and with the theoretical [31,32]. The objective of this training area is to develop teaching competencies aimed at promoting the learning of mathematics in concrete school situations. This area is organized on the basis of four lines of action: planning, execution and evaluation practices of the mathematics teaching and learning process; reflection and analysis of teaching practice; innovation practices and the incorporation of good practices. The future mathematics teacher must develop competencies to plan, guide the process and evaluate it, from a specific class or a learning unit to an entire mathematics course. This goes through an initial process of familiarization with the teaching practice, a process of partial responsibility up to total responsibility for the process. It is based on the principle that the professional is mainly formed in practice, but the incorporation to practice cannot be sudden, but gradual and systematic [33].

On the other hand, it is widely accepted that reflection is one of the fundamental mechanisms for promoting change and professional development of teachers. In fact, reflection on practice is considered the backbone of teaching, capable of integrating the different components and favoring the understanding of one's own teaching in order to improve it. This is precisely the objective pursued by this line of action [22]. The reflection of the future professional can revolve around several elements: on the methodological tools or the teaching method, his or her system of beliefs and values, his or her knowledge of teaching, on the content of teaching, on the learning difficulties of his or her students and the ways to help them overcome them, on the data collected during teaching in the classroom, on the modification of his or her didactic performance according to the conditions of the context, among others. All these reflections can help the future professional to evaluate his or her teaching performance and incorporate any type of information about it for subsequent decision-making with a view to improving teaching [32].

Teacher training cannot remain only at the reproductive level; it is necessary to prepare teachers to change and innovate the teaching of mathematics. Carbonell (2006) considers educational innovation as a series of interventions, decisions and processes, with a certain degree of intentionality and systematization, that seek to modify attitudes, ideas, culture, contents and pedagogical practices, and at the same time, to introduce, in a renovating line, new projects and programs, curricular materials, teaching-learning strategies, didactic models and other ways of organizing and managing the curriculum, the center and the classroom dynamics [7,10].

The ability to innovate the process also involves reflection on practice, always with a view to improving teaching, and this is measured by the effectiveness in the production of student learning. In reality, one changes in order to improve, although in practice this does not necessarily happen. The innovation process, therefore, can be subjected to scientific observation and evaluation criteria. Innovation projects must go through their design, implementation and evaluation in concrete school situations, and from there move on to redesign and subsequent implementation. It is a question of going back and forth from theory to practice and from practice to theory, with the purpose of achieving continuous improvement and enhancement of teaching practice. This is the main objective of this line of action, which implies the knowledge and use of the essential elements of educational research [34,35].

The knowledge and incorporation of good practices to the practice of the future professional is essential for the training of the Mathematics teacher [5,36]. Good practices are conceived as those that achieve the planned learning objectives (Planas and Alsina, 2009); in our case, a good practice is one that manages to promote the learning of mathematics in students. Referring to curricular changes, reforms are carried out involving teachers, that is, from within, preserving the best of the installed practices and seeking "improving" balances between reforms, teachers and student learning, so that improvement has more chances of becoming a reality. In general, good practices have been disdained, believing that the new is always better than the old, or that the experience of teachers is not important. In order to make this line of action more dynamic, several processes are followed: getting to know in situ the successful experiences of mathematics teachers; analyzing and reflecting on these practices; incorporating and reproducing these practices; evaluating and improving them [5,15].

4. Conclusions

This article has addressed issues that, in our opinion, are central to the training of mathematics teachers. Some of these issues are not usually addressed by Mexican researchers in our field, in particular, the one referring to how mathematics teaching professionals should be trained, I am referring to mathematics teachers for elementary and high school education. In general terms, the basic curricular structure of the training is outlined. This is organized on the basis of three training areas: mathematics, pedagogy and teaching. However, there are nowadays different points of view about this type of training that can surely contribute to achieve a more robust training.

Current scenarios such as globalization, the knowledge and information era, the ever-increasing universe of available information and communication, the changes in social coexistence and political participation, and the increasingly demanding characteristics of the labor market, require a mathematics teacher who responds to these demands and those of the future. The current and foreseeable changes demand from a mathematics teacher an attitude of continuous change and updating. Here we only touch on the subject of initial training, but it is also necessary to think about continuous training, or as it is usually said when referring to this subject, the updating of practicing teachers.

Currently, trends in teacher training point towards a better integration of theory and practice, towards a more effective initial training in mathematics that considers school practice, and towards considering research as an important formative dimension. This implies giving greater importance to teacher training as a measure to improve educational practices, to a more effective university mathematics training of future teachers that considers school practice. On the other hand, there is now talk of new teaching competencies, several of which have already been addressed in this paper. In particular, Perrenoud (2007) proposes 10 teaching skills: organizing and animating learning situations; managing learning progression; developing and developing differentiation devices; involving students in their learning and work; working in teams; participating in school management; informing and involving parents; using new technologies; dealing with the duties and ethical dilemmas of the profession; and organizing one's own continuing education. The challenges now facing the training of mathematics teachers have to do with the possibilities of concretizing these competencies in practical conditions.

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