Application of the Finite Element Method (FEM) through GFAS Software to the study of a tunnel

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Abstract: Computational simulation is widely used in companies to perform analysis and improve the quality of products and projects. Most of these analyses are carried out using software that uses the Finite Element Method, which allows to obtain answers to numerous engineering problems. In this study, two examples of application to the study of tunnels of the Finite Element Method using the Geostru Software "GFAS – Geotechnical F.E.M. Analysis System" are proposed. The case of a tunnel excavated inside a granite rock massif was analyzed, first determining the state of stresses in the cavity contour through a theoretical method and comparing these results with those obtained in the software. Then, by means of finite element modeling, the settlements induced by the excavation were determined. Finally, the problem of tunnel excavation in a viscoplastic rock mass is presented and the authors propose a comparison of the analytical and numerical method.

Keywords: tunnels; stresses; FEM; GFAS

Aplicación del método de los elementos finitos (MEF) mediante el software GFAS al estudio de un túnel

Resumen: La simulación computacional se utiliza ampliamente en las empresas para realizar análisis y mejorar la calidad de los productos y proyectos. La mayoría de estos análisis se llevan a cabo mediante programas informáticos que utilizan el Método de
los Elementos Finitos, que permite obtener respuestas a numerosos problemas de ingeniería. En este estudio se proponen dos ejemplos de aplicación al estudio de túneles del Método de los Elementos Finitos utilizando el Software Geostru “GFAS – Geotechnical F.E.M. Analysis System”. Se analizó el caso de un túnel excavado en el interior de un macizo rocoso de granito, determinando primero el estado de tensiones en el contorno de la cavidad mediante un método teórico y comparando estos resultados con los obtenidos en el software. A continuación, mediante la modelización por elementos finitos, se determinaron los asentamientos inducidos por la excavación. Por último, se presenta el problema de la excavación de un túnel en un macizo rocoso viscoplástico y los autores proponen una comparación del método analítico y el numérico.

Palabras claves: túneles; tensiones; FEM; GFAS

1. Introduction

Computational simulation is widely used in companies to perform analysis and improve the quality of products and projects. Most of these analyses are carried out using software that uses the Finite Element Method, which allows to obtain answers to numerous engineering problems [1-4]. The geometry of the part, subjected to loads and constraints, is subdivided into smaller parts, known as "elements", which represent the continuous domain of the problem. Dividing the geometry into small elements solves a complex problem by subdividing it into simpler problems, allowing the computer to perform the tasks efficiently [5-7].

The method proposes that an infinite number of unknown variables be replaced by a limited number of elements of well-defined behavior. These divisions can have different shapes, such as triangular, quadrangular, among others, depending on the type and size of the problem [8]. As the number of elements is limited, they are called "finite element" – the word that gives name to the method. The finite elements are connected to each other by points, which are called nodes or nodal points. The set of all these items – elements and nodes – is called a mesh. Due to the subdivisions of the geometry, the mathematical equations governing the physical behavior will not be solved in an exact but approximate way by this numerical method [9]. Computational simulation software is evolving and improving the analysis based on the method, promoting the improvement of type selection and element mesh generation, modeling techniques, acceptance criteria, errors and presentation of results, allowing easier use of the tools. Therefore, the knowledge of the fundamentals of the method is necessary to develop, together with the software mastery, the best practices for the application of this powerful resource in the development and evaluation of products and projects. In the application of the Finite Element Method, the method can be applied in the resolution and diagnosis of structural analysis problems to obtain displacements, deformations and stresses [10,11].

In the past, the utility of the finite element modeling (FEM) was first recognized in the early 1940s (eg. [12,13]) and it took several decades to apply this approach in various fields. So, finite element methods spread quickly through time assuming a dominant role in geotechnical areas, mining and engineering practice. In our work we want to focus on the use of this methodology (FEM) by applying it to the excavation of a tunnel inside a granite rock mass. Numerical methods constitute a very valuable aid, particularly the finite element method is still the most used model in calculation stresses and displacements for tunnel excavation [14-16]. Preliminarily, in this work, the tangential stresses on the contour of the cavity are evaluated in correspondence with “singular” points by using the method of OBERT et alii [16]. Subsequently, through the use of the Geostru – GFAS software, the same stress states are calculated on the basis of a numerical modeling, in order to be able to compare the results obtained. Finally, again with the aid of the same software, the displacements induced by the excavation are calculated [17,18].
2. Materials and methods

There are various theories to calculate the stress state induced by the excavation of a tunnel. For the present work the OBERT et alii [3] approach was considered. This theory is applicable on “competent” (capacity of sustaining openings without any structural support because of its phisic-geological features), massive, intact and homogeneous rocks and involves the determination of the maximum stresses generated in the rock around a cavity. It is based on a series of simplifying hypotheses, which, in addition to admitting an elastic behavior of the rock mass, describe the distribution of stresses around the cavity as those obtained in a circular opening infinitely far from the boundaries of a plate subjected to a uniform bidirectional stress field (Fig. 1).

![Figure 1. Stress state at the boundary of a circular cavity. From OBERT et alii [3].](image1)

In the Figure 2 is shown the trend of the radial stresses ($\sigma_r$) and tangential stresses ($\sigma_\theta$) along the horizontal and vertical direction with the ratio $r/a$ (where $a$ is the radius of the cavity and $r$ is the distance of the point considered from the center of the cavity), in the hypothesis of unidirectional stress field ($S_h=M; S_v=0$).

![Figure 2. Trend of the radial and tangential stresses (normalized with respect to the vertical tension acting at the height of the center of the cavity in undisturbed conditions) along the vertical and horizontal directions in the hypothesis of unidirectional tension field. From OBERT et alii [3].](image2)
For tunnels with a circular section, the values of the radial stress $\sigma_r$ and the tangential stress $\sigma_\theta$ at any point located on the polar coordinate $\theta$ at the distance $r$ from the center of a cavity with radius $a$, are given by the following expressions:

$$\sigma_r = \frac{(S_h + S_v)}{2} \left( 1 - \frac{a^2}{r^2} \right) - \left( S_h - S_v \right) \left( 1 - 4 \frac{a^2}{r^2} + 3 \frac{a^4}{r^4} \right) \cos 2\theta$$

$$\sigma_\theta = \frac{(S_h + S_v)}{2} \left( 1 + \frac{a^2}{r^2} \right) - \left( S_h - S_v \right) \left( 1 + 3 \frac{a^2}{r^2} \right) \cos 2\theta$$

At the contour of the cavity ($r = a$), the radial stress $\sigma_r$ is zero, while the tangential stress $\sigma_\theta$ in the points of coordinates $\theta = 0^{\circ}$ (on the horizontal) and $\theta = 90^{\circ}$ (on the vertical) assume the following values:

$$\sigma_\theta(\theta=0^\circ) = 3 S_v - S_h; \sigma_\theta(\theta=90^\circ) = 3 S_h - S_v$$

with:

$$S_v = \gamma H; \ S_h = M S_v$$

Where $\gamma$ is the specific weight of the material, $H$ is the height of the covering layers and $M$ represents the lateral thrust coefficient, linked to the Poisson’s ratio $\nu$ by the following relationship:

$$M = \frac{\nu}{(1 - \nu)}$$

In this specific case, the construction of a circular section tunnel with a radius of 5m with a covering height of 20m, dug in a granite rock mass, whose geomechanical characteristics, hypothesized on the basis of data obtained from the literature [4], are shown in Table 1:

<table>
<thead>
<tr>
<th>Table 1. Geomechanical parameters assumed.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific weight $\gamma$ [kg/m$^3$]</td>
</tr>
<tr>
<td>Friction angle $\phi$ [\textdegree]</td>
</tr>
<tr>
<td>Poisson Coefficient $\nu$</td>
</tr>
</tbody>
</table>

Applying the theory of OBERT et alii [3] to the contour of the excavation the values of $\sigma_\theta$ are obtained (Tab. 2).

<table>
<thead>
<tr>
<th>Table 2. Stresses $\sigma_\theta$ around the excavation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 0^\circ$</td>
</tr>
<tr>
<td>$S_v$ [kg/cm$^2$]</td>
</tr>
<tr>
<td>$S_h$ [kg/cm$^2$]</td>
</tr>
<tr>
<td>$\sigma_\theta$ [kg/cm$^2$]</td>
</tr>
</tbody>
</table>

As can be seen, the value of $\sigma_\theta$ at the boundary along the horizontal symmetry axis is much higher than that along the vertical symmetry axis.

By plotting the trend of the radial stress $\sigma_r$ and the tangential stress $\sigma_\theta$ with respect to the distance from the center of the cavity $r$ along the horizontal ($\theta = 0^\circ$), it can be observed that, moving away from the center of the cavity, the value of the tangential stress tends to value of the vertical stress $S_v$, while the value of the radial stress tends towards the horizontal stress $S_h$ value (Fig. 3).
In Figure 4 is reported the trend of the radial stress $\sigma_r$ and the tangential stress $\sigma_\theta$ with respect to the distance from the center of the cavity $r$ along the vertical axis ($\theta = 90^\circ$).

For the application of the finite element methodology a numerical approach was used. In particular, we used GFAS: a finite element software that has been developed specifically for the analysis of deformation and stability analysis in geotechnical engineering problems [4].

To calculate the stresses on the boundary of the excavation using the GFAS software, it is necessary to first create the region that contains the tunnel with a circular section. It then proceeds with the creation of an unstructured calculation mesh and, subsequently, the geometric characteristics of the considered region (fig. 5) and the mechanical characteristics of the granitic rock are inserted.
The quantities required by the software and the relative entered values are summarized in Table 3.

### Table 3. Stresses \( \sigma \) around the excavation.

<table>
<thead>
<tr>
<th>Elastic Module ([kN/m^2])</th>
<th>49033250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson Coefficient</td>
<td>0.3</td>
</tr>
<tr>
<td>Thickness ([m])</td>
<td>1</td>
</tr>
<tr>
<td>Specific weight ([kN/m^3])</td>
<td>27</td>
</tr>
<tr>
<td>(K_0)</td>
<td>0.234</td>
</tr>
</tbody>
</table>

Among the failure criteria available by the software, the Mohr–Coulomb one was chosen which generally recommended for all types of materials, because it is based on a limited number of parameters and, above all, is the most consolidated among the criteria known in literature.

In Table 4 input data are reported.

### Table 4. Input data for the characterization of the material behavior.

<table>
<thead>
<tr>
<th>Cohesion ([kg/m^2])</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction angle (^{\circ})</td>
<td>50</td>
</tr>
<tr>
<td>Dilatancy angle (^{\circ})</td>
<td>0</td>
</tr>
<tr>
<td>Failure Criterion</td>
<td>Mohr–Coulomb</td>
</tr>
</tbody>
</table>

After entering the data relating to the properties of the material, it is necessary to assign the constraints for the mesh creation consisting of a certain number of nodes (Fig. 6).

![Figure 6. FEM Mesh created by GFAS software. The blue arrows indicate the constraints.](image)

Before determining the stresses related to all the nodes of the considered region around the circular cable, in particular for the nodes 37 and 384 (as shown in Figure 7) that belong to geometric
elements, it is necessary to selected on the software the non-linear analysis according to the Von Mises method and to consider a viscoplastic behaviour for the granitic rock (eg. [6,7]).

**Figure 7.** Zoom on the region and identification of the nodes (blue circle) corresponding to $\theta = 0^\circ$ and $\theta = 90^\circ$.

Table 5 shows the results relating to the average stresses at the nodes obtained by the GFAS software for the nodes 37 and 384 corresponding to $\theta = 0^\circ$ and $\theta = 90^\circ$, relatively.

<table>
<thead>
<tr>
<th>$\theta$ [°]</th>
<th>Node</th>
<th>$\sigma$ [kg/cm$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>37</td>
<td>10.5</td>
</tr>
<tr>
<td>90</td>
<td>384</td>
<td>1.94</td>
</tr>
</tbody>
</table>

Observing Figure 8, it is interesting to note that the greatest stress are around the circle, in particular in correspondence with the horizontal axis, where orange and red colour are associated with higher stress values and it indicates a lower strength of the material. This colour distribution is more evident in the element stressed trend (Fig.8A than nodal stressed trend (Fig.8B).

**Figure 8.** Representation of the stresses trend calculated with Von Mises method in GFAS: A) Nodal stresses; B) Element stresses.
4. Discussion

Finally, we compared the results obtained by applying the classical theory (Tab. 2) with those elaborated using the GFAS software (Tab. 6). The obtained data by using the approach proposed by OBERT et alii [16,19], based on the assumption that the rock mass can be assimilated to an isotropic, homogeneous and elastic are in accordance with those calculated by the software.

<table>
<thead>
<tr>
<th>Node</th>
<th>$\theta = 0^\circ$</th>
<th>$\theta = 90^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
<td>10,5</td>
<td>10,73</td>
</tr>
<tr>
<td>38</td>
<td>10,73</td>
<td>10,73</td>
</tr>
<tr>
<td>55</td>
<td>9,44</td>
<td>9,44</td>
</tr>
<tr>
<td>384</td>
<td>1,94</td>
<td>2,16</td>
</tr>
<tr>
<td>388</td>
<td>2,16</td>
<td>2,05</td>
</tr>
<tr>
<td>423</td>
<td>2,05</td>
<td></td>
</tr>
</tbody>
</table>

Table 6. GFAS results.

Note that through the software GFAS the tensions are computed in the integration point, in detail it means that considering an element (triangle finite element with 3 nodes) the integration point is placed in the gravitational center of the element one. Moreover, the stresses and strains are computed in the integration points not in nodes. In order to evaluate the stresses in the finite element nodes an extrapolation procedure is applied [20,21].

The results (Tab. 6) are referred to the values obtained for the nodes which constitute an element. In particular, the nodes 388–384–423 form the element 178 along the vertical axis (Fig. 9), while, along the horizontal axis, the element 430 (Fig. 9) consists of nodes 37–38–55.

![Geometric elements distribution constituting the finite element mesh: A) Element 178; B) Element stresses.](image)

From the examination of the values shown in Table 6 we can deduce that moving away from the contour of the cavity the results are more reliable and in agreement with the results obtained with the OBERT et alii [3].

Finally, through the GFAS software, it is possible to calculate the deformed mesh (Fig. 10) and the vertical displacements induced by the excavation (Fig. 11).
Plotting the displacements, a Gaussian distribution which represents the trend of settlements within the granite mass is obtained.

5. Conclusions

Thanks to the finite element software Geostru “GFAS – Geotechnical and F.E.M. Analysis System” it was possible to evaluate the stress state around a circular cavity within a granite rock mass.

From the comparison between the values derived from the application of the numerical method with those arising from the theoretical method OBERT et alii (1960) we noted that the results are congruent and a a good agreement between theoretical and numerical solution by FEM is obtained. Considering that the tensions are computed in the integration point (placed in the gravitational center of the element) and not directly in nodes in order to evaluate the stresses in the finite element nodes an extrapolation procedure is applied. In fact, the theoretical results were already in agreement with the results for the elements but to carry out a better comparison the nodes results was considered.

Therefore, the software works well in this simple case and its use will be much more useful, simple and effective when a more complex cases will be considered.

Moreover, the software also allows the evaluation of the deformation state and the estimation of settlements induced by excavation inside the rock mass: an operation not possible through the simple application of the theoretical method.


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References


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